2015 Mathematics

Advanced Higher

Finalised Marking Instructions

Part One: General Marking Principles for Mathematics Advanced Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Principal Assessor.

(b) Marking should always be positive ie, marks should be awarded for what is correct and not deducted for errors or omissions.

GENERAL MARKING ADVICE: Mathematics Advanced Higher

The marking schemes are written to assist in determining the "minimal acceptable answer" rather than listing every possible correct and incorrect answer. The following notes are offered to support Markers in making judgements on candidates' evidence, and apply to marking both end of unit assessments and course assessments.

General Marking Principles

These principles describe the approach taken when marking Advanced Higher Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values/algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

In the detailed Marking Instructions which follow, marks are shown alongside the line for which they are awarded. When marking, no comments at all should be made on the script. The total mark for each question should appear in one of the right-hand margins. The following codes should be used where applicable:

✓ - correct; X – wrong; working underlined – wrong;

tickcross – mark(s) awarded for follow-through from previous answer;

^ ^ - mark(s) lost through omission of essential working or incomplete answer;

wavy or broken underline – bad form, but not penalised.

Part Two: Marking Instructions for each Question

Question			Expected Answer/s	Max Mark	Additional Guidance
1			$= {}^5C_0\left(\frac{x^2}{3}\right)^5 + {}^5C_1\left(\frac{x^2}{3}\right)^4\left(\frac{-2}{x}\right)^1 + {}^5C_2\left(\frac{x^2}{3}\right)^3\left(\frac{-2}{x}\right)^2$ $+{}^5C_3\left(\frac{x^2}{3}\right)^2\left(\frac{-2}{x}\right)^3 + {}^5C_4\left(\frac{x^2}{3}\right)\left(\frac{-2}{x}\right)^4 + {}^5C_5\left(\frac{-2}{x}\right)^5$ $= \frac{x^{10}}{243} - \frac{10x^7}{81} + \frac{40x^4}{27} - \frac{80x}{9} + \frac{80}{3x^2} - \frac{32}{x^5}$	**4**	•1 correct unsimplified expansion •2 fully simplified powers of x •3 powers of 3 and binomial coefficients. **OR** powers of −2 correct. •4 Completes and simplifies correctly.

Notes:

1.1 Accept negative powers of *x*.

1.2 Coefficients must be fully processed to simplified fractions and whole numbers.

Question			Expected Answer/s	Max Mark	Additional Guidance
2	a		$\frac{dy}{dx}=\frac{5\left(x^2+2\right)-2x\left(5x+1\right)}{\left(x^2+2\right)^2}$ $=\frac{-5x^2-2x+10}{\left(x^2+2\right)^2}$	3	•[1] For using quotient rule and correct denominator[3]. •[2] correct differentiation for both parts of numerator •[3] simplified form[2].
2	b		$f'\left(x\right)=2e^{2x}\sin^2 3x+e^{2x}.2.3.\sin 3x\ \cos 3x$ $=2e^{2x}\sin 3x\left(\sin 3x+3\cos 3x\right)$ **or** $e^{2x}\left(2\sin^2 3x+3\sin 6x\right)$	3	•[4] evidence of using product rule •[5] first term •[6] second term

Notes:

2.1 Alternative Method

Product rule

$y=\left(5x+1\right)\left(x^2+2\right)^{-1}$

$\frac{dy}{dx}=-\left(5x+1\right)\left(x^2+2\right)^{-2}.2x+5\left(x^2+2\right)^{-1}$

•[1] for evidence of using product rule *and* one term correct.

•[2] for second term

$=\frac{-5x^2-2x+10}{\left(x^2+2\right)^2}$

•[3] simplified fraction

2.2 Where a candidate has a wrong, but factorisable expression in the numerator, factorisation is not required for award of this mark.

2.3 Where terms are the wrong way round, lose •[1]

2.4 $\frac{d}{dx}\left[\left(5x+1\right)\left(x^2+2\right)\right]...=15x^2+2x=10:\text{award}\left[\frac{0}{3}\right]$.

Question			Expected Answer/s	Max Mark	Additional Guidance
3			$s_{20} = 320 = \frac{20}{2}(2a + 19d)$ $\Rightarrow 2a + 19d = 32$ $u_{21} = 37 = a + 20d$ $a + 20d = 37$ $\underline{2a + 40d = 74}$ $21d = 42$ $d = 2$ $a = -3$ $s_{10} = \frac{10}{2}(2a + 9d)$ $= 60$ **OR** $a + 20d = 37$ $s_{21} = 320 + 37 = 357$ $357 = \frac{21}{2}(a + 37)$ $\Rightarrow a = -3$ $d = 2$ $s_{10} = 5(-6 + 18) = 60$	5	•[1] for correct substitution into formula[1]. •[2] for correct substitution into formula[1]. •[3] d[1] •[4] a[1] •[5] s_{10} •[1] $a + 20d = 37$ •[2] for correct substitution into formula[1]. •[3] a •[4] d •[5] s_{10}

Notes:

3.1 Accept correct equations for •[1] and •[2] without explicit statement of general formulae. However, simply stating values for *a* and *d*, is not sufficient, so do not award •[1] or •[2], ie working required.

3.2 Candidates can also obtain two simultaneous equations using S_{20} and S_{21} formulae. One mark each, then follow the above for final 3 marks.

Question			Expected Answer/s	Max Mark	Additional Guidance
4			$x^4+y^4+9x-6y=14$ $4x^3+4y^3\frac{dy}{dx}+9-6\frac{dy}{dx}=0$ $\therefore 4\left(1^3\right)+4\left(2^3\right)\frac{dy}{dx}+9-6\frac{dy}{dx}=0$ $\therefore \frac{dy}{dx}=\frac{-1}{2}$ $y-2=-\frac{1}{2}(x-1)$ eqn. tangent: $2y=-x+5$ **or** $y=-\frac{1}{2}x+\frac{5}{2}$ **or** $2y+x-5=0$ **OR** $\frac{dy}{dx}=\frac{-4x^3-9}{4y^3-6}=\frac{4x^3+9}{6-4y^3}$ at $A(1,2)\ \frac{dy}{dx}=\frac{4+9}{6-32}=\frac{-13}{26}=\frac{-1}{2}$	4	•[1] x terms and constant[3]. •[2] y terms •[3] gradient[2]. •[4] equation[1].

Notes:

4.1 Published form would have •[4] at expanded form, not as marked.

4.2 Rearrangement and explicit statement of $\frac{dy}{dx}$ not required for full marks.

4.3 Where candidate asserts that $\frac{d}{dx}(14)=14$, •[1] not given, but •[2], •[3] and •[4] all possible, leading to $26y = x + 51$ (or equivalent) for $\begin{bmatrix}3\\4\end{bmatrix}$

Question			Expected Answer/s	Max Mark	Additional Guidance
5			Singular when det A = 0 $p\begin{vmatrix}p & 1\\-1 & -1\end{vmatrix}-2\begin{vmatrix}3 & 1\\0 & -1\end{vmatrix}+0\begin{vmatrix}3 & p\\0 & -1\end{vmatrix}=0$ $p(-p+1)-2(-3)=0$ $p^2-p-6=0$ $(p-3)(p+2)=0$ $p=3$ **or** $p=-2$	4	•[1] evidence of any valid method for obtaining det$A$ *and* setting = 0[1]. •[2] expansion or equivalent method by first row •[3] for polynomial •[4] solutions (both)

Notes:

5.1 "= 0" needs to appear for full credit.

Question			Expected Answer/s	Max Mark	Additional Guidance
6			$\ln y = \ln 3^{x^2}$ $\ln y = x^2 \ln 3$ $\frac{1}{y}\frac{dy}{dx} = 2x\ln 3$ $\frac{dy}{dx} = 2x\ln 3 . 3^{x^2}$	3	•[1] evidence of taking logs. •[2] for differentiating correctly. •[3] for $\frac{dy}{dx}$ in terms of x and y.
			OR $u = x^2$ $\therefore y = 3^u$ $\therefore \frac{dy}{du} = 3^u \ln 3$ $\therefore \frac{dy}{dx} = 3^{x^2} \ln 3 . 2x$		•[1] correct substitution into original equation. •[2] for differentiating correctly. •[3] for $\frac{dy}{dx}$ in terms of x and y.
			OR $\ln y = x^2 \ln 3$ $e^{\ln y} = e^{x^2 \ln 3}$ $y = \left(e^{x^2}\right)^{\ln 3}$ $\frac{dy}{dx} = 2x\ln 3 . e^{x^2 \ln 3}$		•[1] evidence of taking logs. •[2] for rearranging correctly. •[3] for differentiating to obtain in terms of x and y.

Notes:

6.1 Accept $\frac{dy}{dx} = \ln 3^{2x} . 3^{x^2}$

Question			Expected Answer/s	Max Mark	Additional Guidance
7			$3066 = 713\times 4 + 214$ $713 = 214\times 3 + 71$ $214 = 71\times 3 + 1$ $1 = 214 - 71\times 3$ $= 214 - 3(713 - 214\times 3)$ $= 214\times 10 - 713\times 3$ $= (3066 - 713\times 4)\times 10 - 713\times 3$ $= 3066\times 10 - 713\times 43$ $p = 10 \qquad q = -43$	**4**	•[1] starting correctly •[2] reach GCD •[3] equates GCD from •[2] *and* evidence of substitution •[4] obtains values of *p* and *q*.
Notes: 7.1 GCD does not need to be explicitly stated. 7.2 *p* and *q* *must* be explicitly stated					
8			$\frac{dx}{dt} = \frac{1}{2}(t+1)^{-\frac{1}{2}}$ $\frac{dy}{dt} = -\text{cosec}^2 t$ $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$ $= -\text{cosec}^2 t \times 2\sqrt{t+1}$ $= -2\sqrt{t+1}.\text{cosec}^2 t$	**3**	•[1] obtain $\frac{dx}{dt}$ •[2] obtain $\frac{dy}{dt}$ 1 •[3] obtain $\frac{dy}{dx}$ in terms of *t* in any correct form.
Notes: 8.1 $-\text{cosec}^2 t$ or equivalent.					

Question			Expected Answer/s	Max Mark	Additional Guidance
9			$\binom{n+2}{3}-\binom{n}{3}$	4	
			$=\frac{(n+2)!}{(n+2-3)!\,3!}-\frac{n!}{(n-3)!\,3!}$		•[1] demonstrates understanding of factorial form *algebraically*[1].
			$=\frac{(n+2)(n+1)n(n-1)!}{(n-1)!3!}-\frac{n(n-1)(n-2)(n-3)!}{(n-3)!\,3!}$		•[2] using property $n! = n(n-1)!$ [1]
			$=\frac{(n+2)(n+1)n}{3!}-\frac{n(n-1)(n-2)}{3!}$		•[3] correctly expressing with common denominator **OR** as a single fraction[1,5].
			$=\frac{n}{3!}\left[(n+2)(n+1)-(n-1)(n-2)\right]$* $=\frac{n}{6}\left(n^2+3n+2-n^2+3n-2\right)$ $=\frac{n}{6}(6n)$ $=n^2$ as required		•[4] simplification to answer, line * (or equivalent) essential.
			OR LHS = 10 – 1 = 9 RHS = 9. So, true for n = 3. Assume true for some $n = k$. ie $\binom{k+2}{3}-\binom{k}{3}=k^2$		•[1] Base case *and* assumptive hypothesis stated.
			$\binom{k+3}{3}-\binom{k+1}{3}$ $=\binom{k+2}{2}+\binom{k+2}{3}-\binom{k}{2}-\binom{k}{3}$ ……		•[3] Using identity $\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1}$ in context.
			$=k^2+\binom{k+2}{2}-\binom{k}{2}$ $=k^2+\frac{(k+2)(k+1)}{2}-\frac{(k)(k-1)}{2}$		•[2] Using property that $n! = n(n-1)!$
			$=k^2+2k+1=(k+1)^2$		•[4] Completes proof, including accurate induction statement.

Question				Expected Answer/s	Max Mark	Additional Guidance
9				**(cont)**		

Notes:

9.1 $\bullet^1 \bullet^2$ awarded wherever they appear, eg as part of an attempted induction proof.

9.2 Must include 10-1 to demonstrate application of understanding of how to process factorials. However, if this is satisfactorily demonstrated later, $\bullet^1$ may then be awarded.

9.3 Although successfully completed in only a few cases, proof by induction may be attempted and marks allocated as above.

9.4 Many attempts at induction are likely to include base case and assumptive hypothesis, but then candidates attempt to prove that $\binom{k+3}{3} - \binom{k+1}{3} = (k+1)^2$. Award max $\left[\frac{3}{4}\right]$ since this approach does not use ,6inductive hypothesis and therefore is not a proof by induction.

9.5 Where candidate starts at this line, all 3 marks may be awarded for being correct so far. However, the lack of working is likely to mean that an incorrect expression here may lose more than one mark.

Question			Expected Answer/s	Max Mark	Additional Guidance
10			$\int x^2e^{4x}dx = \frac{1}{4}x^2e^{4x} - \int \frac{1}{2}xe^{4x}dx$ $= \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \int \frac{1}{8}e^{4x}dx$ $= \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x}$ $\int_0^2 x^2e^{4x} = \left[\frac{1}{4}.4.e^8 - \frac{1}{8}.2e^8 + \frac{1}{32}e^8\right] - \left[\frac{1}{32}\right]$ $= e^8\left(1 - \frac{1}{4} + \frac{1}{32}\right) - \frac{1}{32}$ $= \frac{25}{32}e^8 - \frac{1}{32}$ **or** $\frac{1}{32}\left(25e^8 - 1\right)$	5	•[1] knowing and using integration by parts •[2] appropriate choice for u and v' and correct application •[3] Second application[2,4,6]. •[4] final integral and substitute limits. •[5] exact value[1,5,6].

Notes:

10.1 •[5] only available where working required to obtain value has not been eased. eg must have at least three terms and non-zero value resulting from $x = 0$.

10.2 Where 2nd application 'undoes' first and no further progress: max $\left[\frac{1}{5}\right]$.

10.3 Where candidate asserts that $\int e^{4x}dx = 4e^{4x} \xrightarrow{\text{goes to}} 80e^8 - 128 \approx 238{,}348$ or $e^{4x} \xrightarrow{\text{goes to}} 2e^8 - 2 \approx 5{,}959\cdot9..$ lose •[2] (wrong) and •[5] (eased). Award •[4] only if appropriate substitution to exact values appears.

10.4 For wrong signs in either/both "by parts" operations, award:

$uv+, uv+$ max $\left[\frac{3}{5}\right]$, lose •[1] (wrong) and •[5] (eased).

$uv+, uv-$ max $\left[\frac{3}{5}\right]$, lose •[1] (wrong) and •[5] (eased).

$uv-, uv+$, leading to $\frac{23e^8+1}{32}$ max $\left[\frac{4}{5}\right]$ lose only •[3] (wrong).

10.5 Lose final mark when appropriated to 2328·84… when no exact version.

10.6 Where final integration is subtracted, again leading to $\frac{1}{32}\left(23e^8+1\right)$ lose •[3] or •[4] $\left[\frac{4}{5}\right]$;

$\frac{1}{32}\left(23e^8-1\right)$ lose •[3] •[5] $\left[\frac{3}{5}\right]$ or 2142·59…lose •[3] •[5] $\left[\frac{3}{5}\right]$ or 2142·53… lose •[3] •[4] •[5] $\left[\frac{2}{5}\right]$.

Question			Expected Answer/s	Max Mark	Additional Guidance
11			$M_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $M_3 = M_1 M_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in the line $y = x$	**4**	•[1] for M_1 •[2] for M_2. •[3] for M_3.[1] •[4] correct interpretation[3].

Notes:

11.1 $M_2 M_1$: Incorrect order gives $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ do not award •[3]

11.2 Incorrect order $M_2 M_1$ leading to reflection in the line $y = -x$, award •[4]

11.3 Accept, in isolation, correct description of single transformation, eg "reflection in line through (0, 0) at 45° to positive direction of the x-axis." Simply stating that "x- and y-coordinates swapped" not sufficient.

Question			Expected Answer/s	Max Mark	Additional Guidance
12			Let numbers be $2n - 1$, $2n + 1$, $n \in ℵ$ $(2n+1)^2 - (2n-1)^2$ $= (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$ $= 8n$ which is divisible by 8	**3**	•[1] correct form for any two consecutive odd numbers[1,2]. •[2] correct expressions squared out. •[3] multiple of 8 and communication.

Notes:

12.1 This line may be omitted and awarded where correct form appears in next line.

12.2 In most cases, use of two different letters, ie two odd numbers not necessarily consecutive, leads to at most only •[2] and •[3] being awarded.

Question			Expected Answer/s	Max Mark	Additional Guidance
13	a		$z^2 = (x+iy)^2$ $x^2 + 2ixy - y^2 = \lvert x+iy\rvert^2 - 4$ $x^2 + 2ixy - y^2 = x^2 + y^2 - 4$ $2ixy - y^2 = y^2 - 4$ $2y^2 - 2ixy - 4 = 0$ $2(y^2 - 2) = 0, \quad y = \pm\sqrt{2},$ $2ixy = 0, \quad x = 0$ $z = \pm\sqrt{2}i$	3	•1 writing in form $x + iy$ [3] *and* either LHS or RHS correct. •2 equating real parts *and* solving to obtain y [2] •3 equating imaginary parts and $x = 0$
13	b		$x^2 + 2ixy - y^2 = i(x^2 + y^2) - 4i$ $x^2 - y^2 + 2ixy = (x^2 + y^2 - 4)i$ $2xy = x^2 + y^2 - 4, x^2 - y^2 = 0$ $x^2 + y^2 - 2xy = 4$ $(x-y)^2 = 4$ $x - y = \pm 2, \quad y = \pm x$ leading to $y = -x$ which gives $z = 1 - i$ and $z = -1 + i$.	4	•4 correct expansion in x and y. •5 equating real and imaginary parts [4]. •6 two equations[1]. •7 solutions.

Notes:

13.1 Or obtain one equation and substitute into other.

13.2 Alternatively for two correct equations equating both real and imaginary parts, without further progress, award •2.

13.3 Accept use of *a* and *b (ie a* + i*b)* or other letters, without penalty if used consistently.

13.4 Classify making same mistake in part (b) as in part (a) as a repeated error, so only penalised if eased.

Question			Expected Answer/s	Max Mark	Additional Guidance
14			$g(x)=f(x)+f(-x)$ $g(-x)=f(-x)+f(x)$ $=f(x)+f(-x)=g(x)$ $\therefore$ since $g(-x)=g(x)$ function is even. $h(x)=f(x)-f(-x)$ $h(-x)=f(-x)-f(x)$ $=-f(x)+f(-x)$ $=-[f(x)-f(-x)]$ $=-h(x)$ $\therefore$ since $h(-x)=-h(x)$ function is odd. $g(x)+h(x)=2f(x)$ by adding initial equations $f(x)=\frac{1}{2}g(x)+\frac{1}{2}h(x)$ $\therefore$ Since g even and h odd, $f(x)$ is the sum of an even and an odd functions.	**4**	$\bullet^1$ communicating knowledge of an even and an odd function[2]. $\bullet^2$ showing that $g(x)$ is even. $\bullet^3$ showing that $h(x)$ is odd. $\bullet^4$ correct expression *and* conclusion.

Notes:

14.1 For $\bullet^3$ writing $-[f(x)-f(-x)]$ is not essential.

14.2 Award $\bullet^1$ where statements appear at start of answer or as part of individual 'show thats'. Geometric description acceptable for $\bullet^1$, but needs to be watertight, eg function will be odd if unchanged by 180^o / π rotation about origin and function is even if unchanged by reflection in y-axis [or line x = 0].

14.3 Accept $f(x)=\frac{1}{2}[g(x)+h(x)]$ as bad form without penalty.

Question			Expected Answer/s	Max Mark	Additional Guidance
15	**a**		$u_1 = i + 2j - k$ direction vector $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ $u_2 = -4i + 4j + k$ direction vector $\begin{pmatrix}-4\\4\\1\end{pmatrix}$ $v_1 = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}, \quad v_2 = \begin{pmatrix}-5\\2\\5\end{pmatrix} + \mu\begin{pmatrix}-4\\4\\1\end{pmatrix}$	**2**	•[1] & •[2] for vector equations[1,2,3,6,8].
15	**b**		If they intersect $2+\lambda = -5-4\mu \qquad 4\mu+\lambda = -7$ $4+2\lambda = 2+4\mu \qquad \underline{4\mu-2\lambda = 2}$ $1-\lambda = 5+\mu \qquad \lambda = -3$ $\mu = -1$ $z_1 = 1-(-3) \qquad z_2 = 5+(-1)$ $= 4 \qquad = 4$ Since $z_1 = z_2$, the lines intersect at $(-1,-2,4)$	**4**	•[3] two equations for two parameters •[4] two parameter solutions •[5] for checking third component **in both equations.** •[6] point of intersection[4,].

Question			Expected Answer/s	Max Mark	Additional Guidance
15			$u_1 \times u_2$ to get normal	**4**	
			$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & 2 & -1 \\ -4 & 4 & 1 \end{vmatrix}$ or $\begin{vmatrix} i & j & k & i & j \\ 1 & 2 & -1 & 1 & 2 \\ -4 & 4 & 1 & -4 & 4 \end{vmatrix}$		•[7] correct strategy to find normal.
			$= \boldsymbol{i}(2+4) - \boldsymbol{j}(1-4) + \boldsymbol{k}(4+8)$ $= 6\boldsymbol{i} + 3\boldsymbol{j} + 12\boldsymbol{k}$		•[8] correct processing to obtain vector.
			$6x + 3y + 12z = \begin{pmatrix} 6 \\ 3 \\ 12 \end{pmatrix} . \begin{pmatrix} \text{Point of} \\ \text{intersection} \end{pmatrix}$ $= 36$		•[9] substituting normal vector into an equation of a plane. May also use either of the given points.
			So equation of plane is $6x + 3y + 12z = 36$ **OR** $2x + y + 4z = 12$		•[10] *finding correct value for constant and* correct equation.

Note:

15.1 If written in parametric or symmetric form award •[2] not •[1]. Including their statement at the start of 15b.

15.2 If direction vector and fixed point interchanged in one or both, award •[1], but not •[2].

15.3 Do not penalise use of same parameter at this stage.

15.4 Using same parameter for both equations, leading to $\left(\frac{3}{5}, \frac{6}{5}, \frac{12}{5}\right)$ or (3, 6, 0) max $\left[\frac{1}{4}\right]$.

15.5 For $L_1 : i + 2j - k, L_2 : -4i + 4j + k$ or equivalent, lose •[1] but •[2] available (repeated error.)

15.6 Do not penalise vectors written without underlines.

15.7 Acceptable form, without penalty: $\begin{pmatrix} 2+\lambda \\ 4+2\lambda \\ 1-\lambda \end{pmatrix}$.

Question			Expected Answer/s	Max Mark	Additional Guidance
16			$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=3e^{2x}$ $m^2+2m+10=0$ $m=\frac{-2\pm\sqrt{4-4.1.10}}{2}=-1\pm 3i$ $y=e^{-x}\left(A\cos 3x+B\sin 3x\right)$ **OR** $y=ae^{(-1\pm 3i)x}+Be^{(-1-3i)x}$ **try** $y=Ce^{2x}$ $\frac{dy}{dx}=2Ce^{2x}$ $\frac{d^2y}{dx^2}=4Ce^{2x}$ $4Ce^{2x}+4Ce^{2x}+10Ce^{2x}=3e^{2x}$ $C=\frac{1}{6}$ $y=Ae^{-x}\cos 3x+Be^{-x}\sin 3x+\frac{1}{6}e^{2x}$ $1=A+\frac{1}{6}$, $A=\frac{5}{6}$ $\frac{dy}{dx}=-Ae^{-x}\cos 3x-3Ae^{-x}\sin 3x-Be^{-x}\sin 3x+3Be^{-x}\cos 3x+\frac{1}{3}e^{2x}$ $0=-(A)+3B+\frac{1}{3}$, $\frac{5}{6}-\frac{1}{3}=3B$, $B=\frac{1}{6}$ So particular solution is: $y=\frac{5}{6}e^{-x}\cos 3x+\frac{1}{6}e^{-x}\sin 3x+\frac{1}{6}e^{2x}$	**10**	•[1] correct auxillary equation. •[2] solves correctly[1]. •[3] appropriate complementary function. •[4] particular integral •[5] for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ •[6] for finding C •[7] combine CF and PI for general solution[3]. •[8] value of A. •[9] for differentiating correctly[3]. •[10] value of B *and* statement of final answer[4].

Question			Expected Answer/s	Max Mark	Additional Guidance
16.			**(cont)**		

Note:

16.1 For errors in the solution of the auxiliary equation leading to:

A: Complex conjugates, lose $\bullet^2$, but remainder all available, so max $\left[\frac{9}{10}\right]$.

B: Two real roots, neither of which is 2, lose $\bullet^2$, but $\bullet^{3\text{-}8}$ all available, so max . $\left[\frac{7}{10}\right]$ Lose $\bullet^2\bullet^9\bullet^{10}$.

C: Two real roots, one of which is 2, lose $\bullet^2$, but $\bullet^{3\text{-}8}$ and $\bullet^{10}$ all available, so max $\left[\frac{8}{10}\right]$. Lose $\bullet^2\bullet^9$.

D: Ignores RHS completely, ie treats as homogeneous: max $\left[\frac{5}{10}\right]$. Only $\bullet^1\bullet^2\bullet^3\bullet^9\bullet^{10}$ available ($\bullet^8$ eased, so not available).

16.2 Omitting PI from general solution, lose $\bullet^7$ and $\bullet^8$, but $\bullet^9$ and $\bullet^{10}$ both available, so max $\left[\frac{8}{10}\right]$.

16.3 May award $\bullet^7$ at $\bullet^9$ point if clear that CF and PI have been incorporated to produce GS differentiated.

16.4 May award $\bullet^{10}$ if GS explicitly stated earlier *and* values of *A* and *B* are clearly identified.

Question			Expected Answer/s	Max Mark	Additional Guidance
17			$$\begin{array}{r} 2 \\ x^3-3x^2+x-3\overline{)2x^3+0x^2-x-1} \\ \underline{2x^3-6x^2+2x-6} \\ 6x^2-3x+5 \end{array}$$ $$\int\frac{2x^3-x-1}{(x-3)(x^2+1)}dx=\int\left(2+\frac{6x^2-3x+5}{(x-3)(x^2+1)}\right)dx$$ $$=\int 2+\frac{A}{x-3}+\frac{Bx+C}{x^2+1}dx$$ $$6x^2-3x+5=A(x^2+1)+(Bx+C)(x-3)$$ $x=0 \quad 5=A-3C$ $x=3 \quad 50=10A \quad \Rightarrow A=5$ $C=0$ $x=1 \quad 8=2A-2B-2C$ $8=10-2B \quad \Rightarrow B=1$ $$\int\frac{2x^3-x-1}{(x-3)(x^2+1)}dx=\int 2+\frac{5}{x-3}+\frac{x}{x^2+1}dx$$ $$=2x+5\ln\lvert x-3\rvert+\frac{1}{2}\ln(x^2+1)+k$$	9	•[1] for knowing to divide and starting division •[2] correct division[1]. •[3] for correct form of PFs[5]. •[4] creating correct equation •[5] for any two values[4]. •[6] for third value[4]. •[7] for putting into integral *and* any one term correctly integrated[3]. •[8] for any second term. •[9] for third term and + *k*.[2]

Note:

17.1 Where candidate has NOT carried out division see COWAs (Commonly Occuring Wrong Answers) below.
17.2 Do not penalise slightly ambiguous use of "+C" rather than introducing some new letter.
17.3 Do not penalise (legitimate) omission of |absolute value| symbols.
17.4 For incorrect answers, some evidence of provenance of values for *A*, *B* and *C* is required for the award of BOTH •[5] and •[6].
17.5 Check here that candidate has included +*C* in PFs, since omission will usually lead to the **correct** answer or similar, as *C* = 0. Omission of +*C* means COWA D.

Question			Expected Answer/s	Max Mark	Additional Guidance
17			**COWAs**	**9**	
			If don't ÷		
		A	$\frac{2x^3-x-1}{(x-3)(x^2+1)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+1}$		$\bullet^1,\bullet^2$ not available. $\bullet^3$ available.
			$2x^3-x-1=A(x^2+1)+(x-3)(Bx+C)$ $x=3 \quad 50=10A \quad A=5$ $x=0 \quad -1=A-3C \quad C=2$ $x=1 \quad 0=2A-2B-2C$ $0=10-2B-4 \quad B=3$ WRONG		$\bullet^4$ not available. $\bullet^5$ available[4]. $\bullet^6$ available[5].
			$\int\frac{2x^3-x-1}{(x-3)(x^2+1)}=\int\frac{5}{x-3}+\frac{3x+2}{x^2+1}$ $=5\ln\lvert x-3\rvert+\int\left(\frac{3x}{x^2+1}+\frac{2}{x^2+1}\right)dx$ $=5\ln\lvert x-3\rvert+\frac{3}{2}\ln\lvert x^2+1\rvert+2\tan^{-1}x+k$		$\bullet^7$ available[3]. $\bullet^8$ available. $\bullet^9$ available. Max $\left[\frac{6}{9}\right]$.
		B	If do ÷ then forget to put "2 +"		Lose $\bullet^7$. Max $\left[\frac{8}{9}\right]$.

Question			Expected Answer/s	Max Mark	Additional Guidance
17			**COWAs (cont)**		
		C	$\int 2+\frac{6x^2-3x+5}{(x-3)(x^2+1)}\,dx$		•[1],•[2] available.
			$\frac{6x^2-3x+5}{(x-3)(x^2+1)}=\frac{A}{(x-3)}+\frac{B}{x^2+1}$		•[3] wrong form of PFs.
			$6x^2-3x+5=A(x^2+1)+B(x-3)$ $x=3 \quad 50=10A\Rightarrow A=5$ $x=0 \quad 5=A-3B\Rightarrow B=0$		•[4] available. •[5] available[5]. •[6] not available.
			$\int 2+\frac{6x^2-3x+5}{(x-3)(x^2+1)}\,dx=\int 2+\frac{5}{x-3}\,dx$ $=2x+5\ln\lvert x-3\rvert+k$		•[7] available[3]. •[8] available. •[9] not available. Max $\left[\frac{6}{9}\right]$
		C₂	C without division, leading to: $5\ln\lvert x-3\rvert+2\tan^{-1}x+C$		Only •[5]•[7]•[8]available Max $\left[\frac{3}{9}\right]$
		D	$\frac{6x^2-3x+5}{(x-3)(x^2+1)}=\frac{A}{(x-3)}+\frac{Bx}{x^2+1}$ $6x^2-3x+5=A(x^2+1)+B(x-3)x$ $x=0 \quad 5=A$ $x=3 \quad 50=10A$ $x=1\text{(say)} \quad 8=2A-2B\Rightarrow B=1$		D with division This will usually lead to the **correct** answer or similar, but no consideration of $+C$ leading to $C=0$. Therefore losing •[3] and •[6], so Max $\left[\frac{7}{9}\right]$
		D₂	D without division, leading to a variety of answers.		Only •[5]•[7]•[8]available Max $\left[\frac{3}{9}\right]$

Question			Expected Answer/s	Max Mark	Additional Guidance
18	a		**Method 1** $V = Ah$ (here of below) $\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$ •¹ $\frac{dV}{dh} = A*$ $\therefore \frac{dh}{dV} = \frac{1}{A}*$ $= \frac{1}{A} . -k\sqrt{h}$ •² $= \frac{-k}{A}\sqrt{h}$ **or** **Method 2** $V = Ah$ $\frac{dV}{dt} = \frac{d}{dt}(Ah)$ •¹ $-k\sqrt{h} = A\frac{dh}{dt}$ $\frac{Adh}{dt} = -k\sqrt{h}$ •² $\frac{dh}{dt} = \frac{-k}{A}\sqrt{h}$ **Method 3** $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$ •¹ $-k\sqrt{h} = A\frac{dh}{dt}$ •² **or** **Method 4** $h = \frac{V}{A}$ •¹ $\frac{dh}{dt} = \frac{dV}{dt} \Big/ A$ •²	2	•¹ (Ah) in brackets and/or *A* following line needed for Method 2, since taking *A* out as a constant necessary to illustrate understanding of validity of step. •² One or both of *lines needed for Method 1.

Question			Expected Answer/s	Max Mark	Additional Guidance
18	b		$\frac{dh}{dt} = -0\cdot3\text{cm}/\text{hr}$ when $h = 144$ $-0\cdot3 = -\frac{k}{A}\sqrt{144}$ $\frac{k}{A} = \frac{1}{40} \therefore A = 40k$ $\frac{dh}{dt} = \frac{-k}{A}\sqrt{h}$ $\int \frac{1}{\sqrt{h}}\,dh = \int \frac{-k}{A}\,dt$ **OR** $\int \frac{1}{\sqrt{h}}\,dh = \int -\frac{1}{40}\,dt$ $2\sqrt{h} = \frac{-k}{A}t + c$ $2\sqrt{144} = c \qquad c = 24$ $2\sqrt{h} = \frac{-k}{A}t + 24$ $\sqrt{h} = \frac{-k}{2A}t + 12$ $h = \left(\frac{-k}{2A}t + 12\right)^2$ $h = \left(\frac{-1}{80}t + 12\right)^2$	**4**	•3 Subs in $\frac{dh}{dt} = -0\cdot3$ *and* $h = 144$. Award this mark if substitution appears in part (d). •4 separating variables[3]. •5 integrating correctly. •6 evaluating constant of integration *and* completion

Question			Expected Answer/s	Max Mark	Additional Guidance
18	**c**		$0=\left(-\frac{1}{80}t+12\right)^2$ $-\frac{1}{80}t+12=0$ $t=960$ hours number days $=\frac{960}{24}=40$ days	**2**	•[7] knowing to set correct expression to zero •[8] Processing to obtain number of days [4]
18	**d**		$A=400\pi$ $\frac{k}{A}=\frac{1}{40}$ $k=10\pi$ $h=\left(\frac{-1}{80}.96+12\right)^2$ $\frac{dV}{dt}=-108\pi$ $\therefore$ Rate to vegetation is $\mathbf{108\pi}\ \text{cm}^3/\text{hr}$	**3**	•[9] for finding *k*. •[10] obtaining h or $\sqrt{h}$ •[11] processing to answer *with* interpretation.

Notes:

18.1 $A\ \frac{dV}{dt}=119\cdot5\pi\,\text{cm}^3/\text{hr}$ which comes from taking $t=4$. Do not award •[10].

18.2 Using *h* = 144 in part d leading to 377, do not award •[10] or •[11].

18.3 Do not penalise omission of integration symbols.

18.4 Where candidate has used 144 instead of 0 initially, •[7] lost, but •[8] available if resulting quadratic solved correctly to obtain both *t* = 0 **and** *t* = 1920, discarding *t* = 0 answer *and* converting to 80 days.

[END OF MARKING INSTRUCTIONS]