National
Qualifications
2018

2018 Mathematics

Advanced Higher

Finalised Marking Instructions

General marking principles for Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

	$x^2+5x+7=9x+4$
This is a transcription error and so the mark is not awarded.	$x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

	$x^2+5x+7=9x+4$
This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x-4x+3=0$
	$(x-3)(x-1)=0$
	$x=1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot 3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] start differentiation [1] •[2] apply chain rule and complete differentiation [2,3]	•[1] $\frac{1}{\sqrt{1-(3x)^2}}\times\ldots$ •[2] $\frac{3}{\sqrt{1-9x^2}}$	**2**

Notes:

1. For •[1] do not accept $\frac{1}{\sqrt{1-3x^2}}\times\ldots$ unless subsequently corrected.

2. For •[2] accept eg $\frac{3}{\sqrt{1-(3x)^2}}$ or $\frac{1}{\sqrt{\frac{1}{9}-x^2}}$.

3. For candidates who interpret $\sin^{-1}3x$ as $(\sin 3x)^{-1}$, •[2] is available for $-(\sin 3x)^{-2}\times 3\cos 3x$.

Commonly Observed Responses:

	(b)		•[3] evidence use of quotient rule with denominator and one term of numerator correct •[4] complete differentiation	•[3] $\frac{5(7x+1)e^{5x}\ldots}{(7x+1)^2}$ **OR** $\frac{\ldots-7e^{5x}}{(7x+1)^2}$ •[4] $\frac{5(7x+1)e^{5x}-7e^{5x}}{(7x+1)^2}$	**2**

Notes:

Commonly Observed Responses:

Alternative Method 1 (Product Rule)

•[3] $5e^{5x}(7x+1)^{-1}\ldots$

•[4] $\ldots-7e^{5x}(7x+1)^{-2}$

Alternative Method 2 (Logarithmic differentiation)

•[3] $\ln y=5x-\ln|7x+1|$ **and** $\frac{1}{y}\frac{dy}{dx}=\ldots$

•[4] $\frac{dy}{dx}=y\left(5-\frac{7}{7x+1}\right)$

Question			Generic scheme	Illustrative scheme	Max mark
1.	(c)		•[5] start to differentiate product with one term correct [1] •[6] complete differentiation of product [1] •[7] differentiate remaining terms •[8] express derivative explicitly in terms of x and y [2]	•[5] $\frac{dy}{dx}\cos x+\ldots$ **OR** $-y\sin x+\ldots$ •[6] $\frac{dy}{dx}\cos x\ldots$ **OR** $-y\sin x$ •[7] $+2y\frac{dy}{dx}=6$ •[8] $\frac{dy}{dx}=\frac{6+y\sin x}{\cos x+2y}$	**4**

Notes:

1. •[5] and •[6] are not available where the differentiation of $ycosx$ leads to only one term.
2. •[8] is available only where $\frac{dy}{dx}$ appears more than once after completing differentiation.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ state expression $\bullet^2$ form equation and find one unknown [1] $\bullet^3$ find second unknown and write integral expression [1,2] $\bullet^4$ integrate [3,4]	$\bullet^1$ $\dfrac{3x-7}{x^2-2x-15}=\dfrac{A}{x+3}+\dfrac{B}{x-5}$ $\bullet^2$ $3x-7=A(x-5)+B(x+3)$ **AND** eg $A=2$ $\bullet^3$ $B=1$ **AND** $\displaystyle\int\left(\frac{2}{x+3}+\frac{1}{x-5}\right)dx$ stated or implied by $\bullet^4$ $\bullet^4$ $2\ln\lvert x+3\rvert+\ln\lvert x-5\rvert+c$	4

Notes:

1. $\bullet^2$ and $\bullet^3$ may be awarded where an attempt at factorisation leads to an incorrect linear denominator.
2. At $\bullet^3$ disregard the omission of dx.
3. At $\bullet^4$ disregard the omission of modulus signs.
4. $\bullet^4$ is not available where the constant of integration has either been omitted or first appears after an incorrect logarithmic term.

Commonly Observed Responses:

$\dfrac{3x-7}{x^2-2x-15}=\dfrac{A}{x-3}+\dfrac{B}{x+5}$	do not award $\bullet^1$
$3x-7=A(x+5)+B(x-3)$ **AND** eg $A=\dfrac{1}{4}$	award $\bullet^2$
$B=\dfrac{11}{4}$ **AND** $\displaystyle\int\left(\frac{1}{4(x-3)}+\frac{11}{4(x+5)}\right)dx$ stated or implied by $\bullet^4$	award $\bullet^3$
$\dfrac{1}{4}\ln\lvert x-3\rvert+\dfrac{11}{4}\ln\lvert x+5\rvert+c$	award $\bullet^4$

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•1 state general term [1,2] •2 simplify powers of x **OR** coefficients [1,2] •3 state simplified general term (complete simplification) [1,2,3]	•1 $\binom{9}{r}(2x)^{9-r}\left(\frac{5}{x^2}\right)^r$ •2 $2^{9-r}5^r$ **OR** x^{9-3r} •3 $\binom{9}{r}2^{9-r}5^r x^{9-3r}$	3

Notes:

1. Where candidates write out a full binomial expansion, •1, •2 and •3 are not available unless the general term is identifiable in (b).
2. Candidates who write down $\binom{9}{r}2^{9-r}5^r x^{9-3r}$ with no working receive full marks.
3. •3 is unavailable to candidates who in (a) produce further incorrect simplification subsequent to a correct answer eg $2^{9-r}5^r$ becomes 10^9 or x^{9-3r} becomes x^{6r}.

Commonly Observed Responses:

1. General term has not been isolated.

$$\sum_{r=0}^{9}\binom{9}{r}(2x)^{9-r}\left(\frac{5}{x^2}\right)^r$$

$$= \sum_{r=0}^{9}\binom{9}{r}2^{9-r}5^r x^{9-r}x^{-2r}$$

$$= \sum_{r=0}^{9}\binom{9}{r}2^{9-r}5^r x^{9-3r}$$

Do not award •1. Award •2 and •3.

2. General term has been isolated.

$$\sum_{r=0}^{9}\binom{9}{r}(2x)^{9-r}\left(\frac{5}{x^2}\right)^r$$

$$= \sum_{r=0}^{9}\binom{9}{r}2^{9-r}5^r x^{9-3r}$$

$$= \binom{9}{r}2^{9-r}5^r x^{9-3r}$$

Disregard the incorrect use of the final equals sign. Award •1, •2 and •3.

3. Binomial expression has been equated to general term.

$$\left(2x+\frac{5}{x^2}\right)^9 = \binom{9}{r}(2x)^{9-r}\left(\frac{5}{x^2}\right)^r$$

Disregard the incorrect use of the equals sign. Award •1.

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•4 determine value of r [1] •5 evaluate term [1,2]	•4 $r=3$ •5 672000	**2**

Notes:

1. Where candidates write out a full expansion •4 may be awarded where this is complete and correct at least as far as the required term. •5 may be awarded only where the required term is clearly identified from the expansion.
2. •5 is not available to candidates who interpret the term independent of x as substituting $x=0$.

Commonly Observed Responses:

Binomial expansion as far as required term.

$$\left(2x+\frac{5}{x^2}\right)^9 = 512x^9 + 11520x^6 + 115200x^3 + 672000\ldots$$

or $$\left(2x+\frac{5}{x^2}\right)^9 = \frac{1953125}{x^{18}} + \frac{7031250}{x^{15}} + \frac{11250000}{x^{12}} + \frac{10500000}{x^9} + \frac{6098400}{x^6} + \frac{2520000}{x^3} + 672000\ldots$$

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 state conjugate •2 substitute for $z_1, \overline{z}_2$, expand and apply $i^2 = -1$ 1,2	•1 $\overline{z}_2 = p + 6i$ stated or implied •2 $(2p-18)+(3p+12)i$	2

Notes:

1. At •2 accept $2p+12i+3pi-18$.
2. To award •2 $(2+3i)$ must be multiplied by another complex number.

Commonly Observed Responses:

Candidates find $z_1 z_2$:

$2p+3pi-12i+18$ do not award •1
award •2

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 find value of p	•3 -4	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•1 start process •2 obtain remainder of 17 1 •3 express gcd in terms of 306 and 119 •4 obtain a and b 2,3	•1 $306 = 2\times119 + 68$ $(119 = 1\times68 + 51)$ •2 $68 = 1\times51 + 17$ $(51 = 3\times17)$ •3 $17 = -1\times119 + 2\,(306 - 2\times119)$ •4 $a = 2,\ b = -5$	4

Notes:

1. At •2 the gcd and the final line of working do not have to be stated explicitly.
2. The minimum requirement for •4 is $306 \times 2 + 119 \times (-5) = 17$.
3. Do not accept $306 \times 2 - 119 \times 5 = 17$ where the values of a and b have not been explicitly stated.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ find $\frac{dy}{dt}$	•¹ $\frac{dy}{dt}=\frac{3}{3t+2}$	5
			•² complete differentiation and relate derivatives [1,2]	•² $\frac{dx}{dt}=2t$ and $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ stated or implied at •³	
			•³ evaluate gradient [2]	•³ $\frac{dy}{dx}=-\frac{9}{2}$	
			•⁴ find coordinates	•⁴ $x=\frac{10}{9}$, $y=0$	
			•⁵ state equation of tangent [3]	•⁵ $y=-\frac{9}{2}x+5$	

Notes:

1. Evidence for •² could include eg $\frac{dy}{dx}=\frac{\frac{3}{3t+2}}{2t}$.
2. Where candidates evaluate $\frac{dx}{dt}$ and $\frac{dy}{dt}$ before finding $\frac{dy}{dx}$, award •² for evaluating the individual derivatives and award •³ for evaluating $\frac{dy}{dx}$.
3. At •⁵ accept eg $y+\frac{9}{2}x-5=0$, $2y+9x=10$. Do not accept $y-0=\ldots$ or $y=-\frac{9}{2}\left(x-\frac{10}{9}\right)$.

Commonly Observed Responses:

Incorrect expression for $\frac{dy}{dt}$.

$\frac{dy}{dt}=\frac{1}{3t+2}$ leading to $y=-\frac{3}{2}x+\frac{5}{3}$ may be awarded a maximum of 4/5.

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] state transpose of C •[2] obtain matrix	•[1] $\begin{pmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$ stated or implied by •[2] •[2] $2C'-D=\begin{pmatrix} -5 & 1 & 0 \\ -k-1 & -2 & -2 \\ 3 & -1 & -3 \end{pmatrix}$	**2**

Notes:

Commonly Observed Responses:

$2C-D=\begin{pmatrix} -5 & 1 & 2 \\ -k-1 & -2 & -2 \\ 1 & -1 & -3 \end{pmatrix}$ award •[2] only

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•[3] begin to find determinant •[4] simplify expression	•[3] $-(k+3)\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}+0\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}-2\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$ •[4] $k+3$	**2**

Notes:

Commonly Observed Responses:

Expansion about the first row

$$1\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix}-1\begin{vmatrix} k+3 & 2 \\ 1 & 1 \end{vmatrix}+2\begin{vmatrix} k+3 & 0 \\ 1 & 1 \end{vmatrix}$$

$$= \quad 1(0-2)-1(k+3-2)+2(k+3-0)$$

$$= \quad -2-k-1+2k+6$$

$$= \quad k+3$$

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	•[5] state value of k	•[5] -3	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 differentiate •2 find limits for u 3 •3 rewrite integral 1,2 •4 integrate and evaluate 4,5,6,7	•1 $\frac{du}{d\theta} = \cos\theta$ •2 $u = \frac{1}{2},\ u = 1$ •3 $2\int_{1/2}^{1} u^4\, du$ •4 $\frac{2}{5}\left[u^5\right]_{1/2}^{1}$ and $\frac{31}{80}$ (or 0·3875)	4

Notes:

1. •3 is available where candidates either omit limits or retain limits for θ.
2. Where candidates attempt to integrate an expression containing both u and θ, where θ is either inside the integrand or erroneously taken outside as a constant, only •1 and •2 may be available.
3. Where candidates do not change limits but who produce working leading to $\frac{2}{5}\left[\sin^5\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$, •2 may be awarded.
4. For candidates who arrive at $\frac{2}{5}\left[\sin^5\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ by inspection full marks are still available.
5. For candidates who integrate incorrectly, •4 may be available provided division by zero does not occur.
6. •4 is not available to candidates who write limits using degrees.
7. At •4 accept decimal answers rounded to at least 3 significant figures.

Commonly Observed Responses:

Integration by parts

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\, d\theta = \left[2\sin^4\theta\sin\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 8\sin^3\theta\cos\theta\sin\theta\, d\theta$$ •1 •2

$$= \left[2\sin^5\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\, d\theta$$

$$5\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\, d\theta = \left[2\sin^5\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$ •3

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\sin^4\theta\cos\theta\, d\theta = \frac{31}{80}$$ •4

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•1 form the sum of three consecutive integers [1,2,3,4,5] •2 communication [1,5]	•1 $(n-1)+n+(n+1)$ •2 $3n$ which is divisible by 3	2

Notes:

1. Candidates may form the sum $n+(n+1)+(n+2)$ leading to $3(n+1)$ at •2.
2. Withhold •1 where candidates construct an expression of the form $an+(an+1)+(an+2)$, where $a \neq \pm 1$ and $a \in \mathbb{Z}$. •2 may be available.
   eg $4n+(4n+1)+(4n+2)$ leading to $3(4n+1)$ which is divisible by 3.
3. Where candidates equate an expression for the sum of 3 consecutive integers to a multiple of 3 see commonly observed responses.
4. At •1 accept an expression such as $n+n+1+n+2$.
5. Withhold •1 and •2 where candidates form one (or more) sum of 3 specific consecutive numbers eg $2+3+4$.

Commonly Observed Responses:

A. $x+(x+1)+(x+2)=3k$

$3x+3=3k$

$3(x+1)=3k$, where $k=x+1$ and $3k$ is divisible by 3.

Award 2/2

B. $x+(x+1)+(x+2)=3k$

$3x+3=3k$

$3(x+1)=3k$, where $k=x+1$.

The candidate has defined k but has not communicated divisibility. Award •1 but not •2.

C. $(k+1)+(k+2)+(k+3)=3k+6$

$\frac{3k+6}{3}=k+2$ therefore statement is true

The candidate has explicitly divided and there is no requirement to state that $k+2$ is an integer. Award 2/2.

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		$\bullet^3$ appropriate form for odd number, decomposed into two consecutive integers [1,2,3]	$\bullet^3$ $2k+1=k+(k+1)$, $k\in\mathbb{Z}$	**1**

Notes:

1. Where candidates write down $2k+1=k+(k+1)$ and omit $k\in\mathbb{Z}$, award $\bullet^3$.
2. Where candidates omit brackets and write down $k + k + 1$ do not award $\bullet^3$ unless the candidate demonstrates that k and $k + 1$ are two consecutive integers eg writing k, $k + 1$.
3. Where candidates begin with consecutive integers, $\bullet^3$ may be awarded only where $2k + 1$ is associated with any odd integer and not by a restatement of the assertion in the question.

Commonly Observed Responses:

A. $k+(k+1)=2k+1$ odd

Do not award $\bullet^3$. The candidate has not defined a general odd integer.

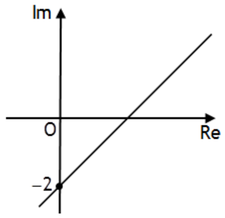
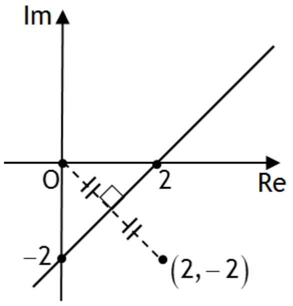
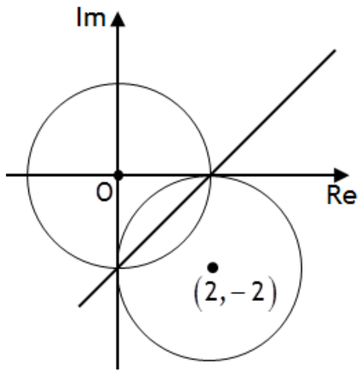
B. $k+(k+1)=2k+1$ and $2k + 1$ represents ***any*** odd integer.

Award $\bullet^3$. The candidate has defined a general odd integer.

C. $2k + 1$

$k+(k+1)=2k+1$

Award $\bullet^3$. The candidate has begun with the general form for an odd integer.

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] substitute, collect real and imaginary parts and equate moduli •[2] process to obtain a linear equation in x and y •[3] sketch consistent with equation [1,2]	•[1] $\lvert x+iy\rvert=\lvert(x-2)+(y+2)i\rvert$ •[2] eg $y=x-2$ •[3] complete sketch 	3
			OR •[1] interpret equation •[2] begin sketch of locus •[3] complete annotations [1,2]	**OR** •[1] line connecting $(0,0)$ and $(2,-2)$ •[2] A straight line exhibiting any one of: bisection, perpendicularity, $(0,-2)$ or $(2,0)$ •[3] A straight line exhibiting any two of: bisection, perpendicularity, $(0,-2)$ or $(2,0)$ 	
			OR •[1] interpret separate loci [3] •[2] interpret conditions •[3] identify required locus [1,2]	**OR** •[1] two circles, centres $(0,0)$ and $(2,-2)$ •[2] circles are congruent and intersect •[3] common chord extended beyond intersection points 	

Question	Generic scheme	Illustrative scheme	Max mark

Notes:

1. At $\bullet^3$ accept any line passing through an appropriate point, which has positive gradient.
2. Do not withhold $\bullet^3$ for axes which are unlabelled. Accept x and y in lieu of 'Re' and 'Im'. Disregard any appearance of i in the diagram.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•¹ obtain A [1]	•¹ $\begin{pmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$	**1**

Notes:

1. Accept $=\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² obtain B	•² $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•³ correct order for multiplication ($P = BA$)	•³ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$	**2**
			•⁴ multiplication completed and appearance of exact values [1,2]	•⁴ $\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$	

Notes:

1. Common factor not required for •⁴.

2. •⁴ is unavailable to candidates who have incorrectly identified $B=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Commonly Observed Responses:

Incorrect order of multiplication

$$\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

Do not award •³. •⁴ is still available on follow through.

Question			Generic scheme	Illustrative scheme	Max mark
	(d)		•5 valid explanation [1,2,3,4]	•5 eg compare the elements of P with the general form of a rotation matrix	1

Notes:

1. •5 may be awarded where a candidate's explanation makes reference to the specific entries of the leading diagonal of P ("they should be equal but are not") or trailing diagonal ("one must be negative of the other but is not")
2. Withhold •5 for a statement such as 'Matrix P represents a reflection' unless the reflection is specified eg 'Matrix P is a reflection in the line $y=-\frac{1}{\sqrt{3}}x$'.
3. •5 may also be awarded where candidates investigate the images of at least two points, neither of which is O.
4. Candidates who respond with reference to the constituent transformations of P may be awarded •5 only where there is reference to both transformations and communication demonstrates understanding that an even number of reflections are required in order to produce a rotation.

Commonly Observed Responses:

Examples of responses where •5 would be awarded

- "P is not associated with rotation about the origin as it is in the form $\begin{pmatrix}\cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta\end{pmatrix}$".

  (Explanation makes explicit reference to their form of P and implicitly refers to the form of a general rotation matrix.)
- "Because P can no longer be expressed as a certain angle of rotation. $\cos^{-1}\left(\frac{1}{2}\right)\neq\cos^{-1}\left(-\frac{1}{2}\right)\therefore$ cannot be expressed as an angle".
- "$\begin{pmatrix}\cos^{-1}\left(\frac{1}{2}\right) & -\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \\ \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) & \cos^{-1}\left(-\frac{1}{2}\right)\end{pmatrix}=\begin{pmatrix}\frac{\pi}{3} & \frac{\pi}{3} \\ -\frac{\pi}{3} & \frac{2\pi}{3}\end{pmatrix}$ and as the angles found are not all equal, this transformation is not a rotation about the origin".

Examples of responses where •5 would not be awarded

"As matrix P doesn't exhibit $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$. (No reference made to form of P).
"Because it gets reflected in the process …".
"Since it is followed by a reflection".
"cos values different so do not relate to same angle".
"Rotation is around a point on graph not the origin".
"It would put the point back to where it started".
"Rotations must have the matrix $\begin{pmatrix}a & -b \\ b & a\end{pmatrix}$".

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.			•[1] show true for $n=1$ [1]	•[1] LHS: $3^0=1$ RHS: $\frac{1}{2}(3-1)=1$ So true for $n=1$	**5**
			•[2] assume (statement) true for $n=k$ **AND** consider whether (statement) true for $n=k+1$ [2,7]	•[2] Suitable statement **and** $\sum_{r=1}^{k}3^{r-1}=\frac{1}{2}\left(3^k-1\right)$ **AND** $\sum_{r=1}^{k+1}3^{r-1}=\ldots$	
			•[3] correct statement for sum to $(k+1)$ terms using inductive hypothesis [3]	•[3] $\ldots=\frac{1}{2}\left(3^k-1\right)+3^{(k+1)-1}$	
			•[4] combine terms in 3^k [4]	•[4] $\frac{3}{2}\times3^k-\frac{1}{2}$	
			•[5] express sum explicitly in terms of $(k+1)$ **or** achieve stated aim/goal **AND** communicate [5,6,7]	•[5] $\frac{1}{2}\left(3^{(k+1)}-1\right)$ If true for $n=k$ then true for $n=k+1$. Also shown true for $n=1$ therefore, by induction, true for all positive integers n.	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

1. "RHS = 1 , LHS = 1" and/or "True for $n = 1$" are insufficient for the award of $\bullet^1$. A candidate must demonstrate evidence of substitution into both expressions.

2. For $\bullet^2$ acceptable phrases for $n = k$ contain:
   - "If true for..."; "Suppose true for…"; "Assume true for…".

   For $\bullet^2$ *unacceptable* phrases for $n = k$ contain:
   - "Consider $n = k$ ", "assume $n = k$ " and "True for $n = k$ ".

   An acceptable phrase may appear at $\bullet^5$.

   For $\bullet^2$, in addition to an acceptable phrase containing $n = k$, accept:
   - "Aim/goal: $\sum_{r=1}^{k+1} 3^{r-1} = \frac{1}{2}\left(3^{k+1} - 1\right)$".

   For $\bullet^2$ *unacceptable* phrases for $n = k + 1$ contain:
   - "Consider true for $n = k + 1$ ", "true for $n = k + 1$ " ;
   - " $\sum_{r=1}^{k+1} 3^{r-1} = \frac{1}{2}\left(3^{k+1} - 1\right)$ " (with no further working)

3. At $\bullet^3$ accept $\ldots = \frac{1}{2}\left(3^k - 1\right) + 3^k$ or $\ldots = \frac{1}{2}\left(3^k - 1\right) + 3^{k+1-1}$.

4. At $\bullet^4$ accept $\ldots = \frac{1}{2}\left(3 \times 3^k - 1\right)$.

5. $\bullet^5$ is unavailable to candidates who write down the correct expression without algebraic justification.

6. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently achieve the stated aim/goal.

7. Following the required algebra and statement of the inductive hypothesis, the minimum acceptable response for $\bullet^5$ is "Then true for $n = k + 1$, but since true for $n = 1$, then true for all n" or equivalent.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ Determine the relationship between x and h ¹	•¹ $x^2+h^2=2500$ $h=\sqrt{2500-x^2}$	**1**

Notes:

1. Refer to general marking principle (m).

Commonly Observed Responses:

$\left(\frac{1}{2}h\right)^2=25^2-\left(\frac{1}{2}x\right)^2$	leading to correct answer	award •¹
$\frac{1}{2}h^2=25^2-\frac{1}{2}x^2$	leading to $h^2=50^2-x^2$	do not award •¹
$\frac{1}{2}h^2=25^2-\frac{1}{2}x^2$	leading to $h^2=4\times25^2-x^2$ or equivalent	award •¹
$\frac{1}{2}h=\sqrt{25^2-\frac{1}{4}x^2}$	moving directly to correct answer	award •¹

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² interpret rate of change of x	•² $\frac{dx}{dt}=-0\cdot3$	**5**
			•³ find $\frac{dh}{dx}$	•³ $\frac{dh}{dx}=-2x\cdot\frac{1}{2}\left(2500-x^2\right)^{-\frac{1}{2}}$	
			•⁴ form relationship ⁶	•⁴ $\frac{dh}{dt}=\frac{dh}{dx}\cdot\frac{dx}{dt}$ stated or implied at •⁵	
			•⁵ multiply by $\frac{dx}{dt}$ 1,2,6	•⁵ $\frac{dh}{dt}=\frac{0\cdot3x}{\sqrt{2500-x^2}}$	
			•⁶ evaluate $\frac{dh}{dt}$ 3,4,5,6	•⁶ $\frac{dh}{dt}=\frac{9}{40}\text{cms}^{-1}$	

Notes:

1. At •⁵ candidates may evaluate the derivatives separately eg $\frac{dh}{dt}=-0\cdot75\times(-0\cdot3)$.
2. At •⁵ simplification is not required.
3. At •⁶ units are required. Accept decimal equivalent (0·225 cms^{-1}).
4. Where candidates produce an incorrect answer, accept a decimal rounded to at least 2 significant figures.
5. Award •⁶ only where a candidate's final answer for $\frac{dh}{dt}$ is opposite in sign to that of $\frac{dx}{dt}$.
6. For candidates who do not show evidence of related rates, •⁴, •⁵ and •⁶ are not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)	(i)	$\bullet^1$ multiply first term by a power of the common ratio [1,2,4] $\bullet^2$ find term [3,4]	$\bullet^1$ $80\left(\frac{1}{3}\right)^{\cdots}$ $\bullet^2$ $\frac{80}{729}$	**2**

Notes:

1. At $\bullet^1$ accept any integer index other than 0 or 1.
2. Where candidates elect to repeatedly multiply, the minimum acceptable response for $\bullet^1$ is $80\times\frac{1}{3}\times\frac{1}{3}\ldots$.
3. At $\bullet^2$ accept 0·11.
4. Award full marks for a correct answer with no working.

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	$\bullet^3$ substitute [1,2,3] $\bullet^4$ find sum to infinity [1,2,3]	$\bullet^3$ $\frac{80}{1-\frac{1}{3}}$ $\bullet^4$ 120	**2**

Notes:

1. A correct answer without working receives no credit.
2. $\bullet^3$ and $\bullet^4$ are available only where a formula has been used.
3. At $\bullet^3$ candidates may use either the formula for the sum to infinity or apply a limiting argument using the formula for the sum to n terms (of a geometric progression).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)	(i)	•[5] substitute •[6] find common difference [1]	•[5] eg $\frac{5}{2}\left(2\times 80+(5-1)d\right)=240$ •[6] -16	2

Notes:

1. For a correct answer without working award 0/2.

Commonly Observed Responses:

		(ii)	•[7] find simplified expression	•[7] $96-16n$	1

Notes:

Commonly Observed Responses:

	(c)		•[8] set up equation •[9] obtain quadratic equation in general form [1] •[10] find values of n [2]	•[8] $\frac{n}{2}\left[160+(n-1)(-16)\right]=144$ •[9] $16n^2-176n+288=0$ •[10] $n=2,\ n=9$	3

Notes:

1. At •[9] '$=0$' must appear.
2. At •[10] candidates must obtain 2 positive integer solutions.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•¹ start integration by parts 1,2,3,4,6	•¹ $-\frac{x}{3}\cos 3x - \ldots$	**3**
			•² complete integration by parts 1,2,3,4,6	•² $\ldots \int -\frac{1}{3}\cos 3x\,dx$	
			•³ complete integration 1,2,3,4,5,6	•³ $= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$	

Notes:

1. When integrating, candidates who repeatedly multiply by 3 cannot be awarded •¹ but •² and •³ may still be available.
2. Candidates who communicate an intention to integrate $\sin 3x$ and differentiate x but who inadvertently produce the derivatives of both $\sin 3x$ and x cannot be awarded •¹ but •² and •³ are still available.
3. For candidates who choose $\sin 3x$ as the function to differentiate and x as the function to integrate, •¹ is available only where this is processed correctly. •² and •³ are not available.
4. An error in sign when differentiating or integrating a trigonometric function should not be treated as a repeated error.
5. Do not withhold •³ for the omission of the constant of integration.
6. The evidence for •¹, •² and •³ may appear in (b).

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)		•⁴ identify integral form of integrating factor [1,2]	•⁴ $e^{\int -\frac{2}{x}dx}$	**7**
			•⁵ determine integrating factor [3,6]	•⁵ $\frac{1}{x^2}$	
			•⁶ begin solution	•⁶ $\frac{d}{dx}\left(\frac{1}{x^2}y\right)=\frac{1}{x^2}\left(x^3\sin 3x\right)$ stated or implied at •⁷	
			•⁷ rewrite as integral equation	•⁷ $\frac{1}{x^2}y=\int x\sin 3x\,dx$	
			•⁸ integrate [4,5,6]	•⁸ $\frac{1}{x^2}y=-\frac{x}{3}\cos 3x+\frac{1}{9}\sin 3x+c$	
			•⁹ evaluate constant [4,6,7,8]	•⁹ $c=-\frac{\pi}{3}$	
			•¹⁰ form particular solution [4,6,7,8]	•¹⁰ $y=-\frac{x^3}{3}\cos 3x+\frac{x^2}{9}\sin 3x-\frac{\pi x^2}{3}$	

Notes:

1. Candidates who attempt to solve the equation using eg separation of variables or second order method receive 0/7.
2. Candidates who attempt to apply integration by parts to the entire differential equation receive 0/7.
3. At •⁵ accept an unsimplified integrating factor eg $e^{-2\ln x}$.
4. For candidates who omit the constant of integration, •⁸, •⁹ and •¹⁰ are not available.
5. •⁸ is available only where a candidate integrates correctly based on their RHS at •⁷.
6. Where candidates obtain an integrating factor which is a constant, •⁵, •⁹ and •¹⁰ are not available.
7. For candidates who proceed from •⁸ by multiplying through by x^2: award •⁹ for multiplying through by x^2 and award •¹⁰ for evaluating the constant of integration **then** stating the particular solution.
8. For candidates who proceed from •⁸ by multiplying through by x^2 but who fail to multiply the constant of integration, •⁹ and •¹⁰ are not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•[1] set up augmented matrix	•[1] $\left[\begin{array}{ccc\|c} 1 & -2 & 1 & -4 \\ 3 & -5 & -2 & 1 \\ -7 & 11 & a & -11 \end{array}\right]$	4
			•[2] obtain two zeros [1]	•[2] $\left[\begin{array}{ccc\|c} 1 & -2 & 1 & -4 \\ 0 & 1 & -5 & 13 \\ 0 & -3 & a+7 & -39 \end{array}\right]$	
			•[3] complete row operations [1,2]	•[3] $\left[\begin{array}{ccc\|c} 1 & -2 & 1 & -4 \\ 0 & 1 & -5 & 13 \\ 0 & 0 & a-8 & 0 \end{array}\right]$	
			•[4] obtain value for a [3]	•[4] $a = 8$	

Notes:

1. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •[2] and •[3].
2. For •[3] accept any equivalent form.
3. •[4] is not available unless the candidate's augmented matrix exhibits redundancy.

Commonly Observed Responses:

	(b)		•[5] introduce parameter and substitute [1,2]	•[5] $z = t$, $y - 5t = 13$	2
			•[6] equation of line [1,3]	•[6] $x = 22 + 9t$, $y = 13 + 5t$, $z = t$	

Notes:

1. •[5] and •[6] are not available for substituting in either a numerical value or any expression in terms of a.
2. •[5] is not available where the candidate substitutes into a row containing two other variables.
3. For •[6] accept symmetric or vector form.

Commonly Observed Responses:

For the line, $\mathbf{d} = \mathbf{n}_{\pi_1} \times \mathbf{n}_{\pi_4} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 1 \end{pmatrix}$.

eg Let $z = 0$ so that $x - 2y = -4$. Award •[5]

Form second equation eg $3x - 5y = 1$ and solve
to give $x = 22$, $y = 13$ leading to $\dfrac{x-22}{9} = \dfrac{y-13}{5} = \dfrac{z}{1}\ (= \lambda)$. Award •[6]

Question			Generic scheme	Illustrative scheme	Max mark
16.	(c)		•7 write down normals [1,4]	•7 $\begin{pmatrix}1\\-2\\1\end{pmatrix}$, $\begin{pmatrix}-3\\5\\2\end{pmatrix}$ stated or implied	**3**
			•8 start to find angle	•8 $\cos\theta = \dfrac{-11}{\sqrt{38}\sqrt{6}}$ **OR** $\cos\theta = \dfrac{11}{\sqrt{38}\sqrt{6}}$	
			•9 find acute angle [2,3,5]	•9 0·75	

Notes:

1. At •**7** accept the use of $\begin{pmatrix}-9\\15\\6\end{pmatrix}$.
2. Accept an answer in degrees which rounds to 43°.
3. •**9** is not available for incorrect working subsequent to a correct answer eg 90° – 43°.
4. At •**7** accept eg $\pi_1 = \begin{pmatrix}1\\-2\\1\end{pmatrix}$ or $\pi_4 = \begin{pmatrix}-3\\5\\2\end{pmatrix}$ but not at •**10**.
5. For candidates who express an answer in degrees, the degree symbol must appear.

Commonly Observed Responses:

Solution obtained by rearrangement of the vector product formula

$\sin\theta = \dfrac{\sqrt{107}}{\sqrt{6}\sqrt{38}}$ award •8

Question			Generic Scheme	Illustrative Scheme	Max Mark
16.	(d)		$\bullet^{10}$ explanation [1,2,3]	$\bullet^{10}$ Planes π_2 and π_4 are parallel because the normal of π_4 is a multiple of the normal of π_2 .	**1**

Notes:

1. For the award of $\bullet^{10}$ a statement must compare normal vectors or coefficients of x, y and z.

   Accept eg $\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = -3\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$ or 'The normals are multiples of one another' as justification for the planes being parallel.
2. Do not accept a plane equating to a vector eg $\pi_2 = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix}$.
3. Withhold $\bullet^{10}$ from candidates who provide a correct description but who subsequently write eg $\pi_4 = -3\pi_2$ or make reference to "direction vectors".

Commonly Observed Responses:

The planes are parallel because:

1.	their normals are multiples of each other.	Award $\bullet^{10}$
2.	$\pi_4 = -3\pi_2$.	Do not award $\bullet^{10}$
3.	their direction vectors are multiples of each other.	Do not award $\bullet^{10}$

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		**Method 1**	**Method 1**	**2**
			$\bullet^1$ first derivative and two evaluations **OR** all three derivatives **OR** all four evaluations	$\bullet^1$ $f(x)=e^{2x}$ $\quad f(0)=1$ $f'(x)=2e^{2x}$ $\quad f'(0)=2$ $f''(x)=4e^{2x}$ $\quad f''(0)=4$ $f'''(x)=8e^{2x}$ $\quad f'''(0)=8$	
			$\bullet^2$ obtain expression [1]	$\bullet^2$ $f(x)=1+2x+2x^2+\frac{4}{3}x^3...$	
			Method 2	**Method 2**	
			$\bullet^1$ write down Maclaurin series for e^x	$\bullet^1$ $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}...$	
			$\bullet^2$ substitute [1]	$\bullet^2$ $f(x)=1+2x+2x^2+\frac{4}{3}x^3...$	

Notes:

1. Simplification might not appear until (c)

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>17.</td><td>(b)</td><td>(i)</td><td>•³ find $g''(x)$

•⁴ evidence of product rule [1,2]

•⁵ complete proof [3,4]</td><td>•³ $g''(x)=2\sec x\sec x\tan x$

•⁴ $g'''(x)=2\sec^2 x(\ldots)+(\ldots)\tan x$

•⁵ $g'''(x)=2\sec^2 x\left(\sec^2 x\right)+\left(4\sec^2 x\tan x\right)\tan x$</td><td>3</td></tr>
<tr><td colspan="6">Notes:
1. Candidates can be awarded •⁴ only where the product or quotient rule is required to differentiate their expression for $g''(x)$.
2. At •⁴ there must be clear evidence of the product rule (or quotient rule).
3. •⁵ is not available to candidates who obtain an incorrect answer at •³.
4. •⁵ can be awarded only where the candidate completes the differentiation correctly and shows clearly that the result is equivalent to the expression asked for in the question.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•⁶ completes ALL evaluations

•⁷ substitute [1]</td><td>•⁶ $g(0)=0$
$g'(0)=1$
$g''(0)=0$
$g'''(0)=2$

•⁷ $g(x)=x+\frac{1}{3}x^3\ldots$</td><td>2</td></tr>
<tr><td colspan="6">Notes:
1. •⁷ is available only for powers of x with numerical coefficients.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
17.	(c)		•[8] multiply expressions [1] •[9] multiply out and simplify [2]	•[8] $\left(1+2x+2x^2+\ldots\right)\left(x+\frac{1}{3}x^3\ldots\right)$ •[9] $x+2x^2+\frac{7}{3}x^3\ldots$	**2**

Notes:

1. For candidates who proceed via differentiation •**[8]** is available for obtaining all three derivatives correctly.
2. •[9] is available only for powers of x with numerical coefficients.

Commonly Observed Responses:

	(d)		•[10] write down terms	•[10] $1+4x+7x^2$	**1**

Notes:

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]