X100/701

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**

Marks

Answer all the questions.

1. (*a*) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of x for which $f'(x) = 0$. **3**

(*b*) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$. **4**

2. Given the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.

(*a*) Find A^{-1} in terms of t when A is non-singular. **3**

(*b*) Write down the value of t such that A is singular. **1**

(*c*) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t. **1**

3. Given that

$$x^2 e^y \frac{dy}{dx} = 1$$

and $y = 0$ when $x = 1$, find y in terms of x. **4**

4. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}.$$ **5**

5. Show that

$$\int_{\ln\frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx = \ln\frac{9}{5}.$$ **4**

6. Express $z = \frac{(1+2i)^2}{7-i}$ in the form $a + ib$ where a and b are real numbers.

Show z on an Argand diagram and evaluate $|z|$ and $\arg(z)$. **6**

Marks

7. Use the substitution $x = 2\sin\theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$. **6**
(Note that $\cos 2A = 1 - 2\sin^2 A$.)

8. (*a*) Write down the binomial expansion of $(1 + x)^5$. **1**

(*b*) Hence show that $0{\cdot}9^5$ is $0{\cdot}59049$. **2**

9. Use integration by parts to obtain the exact value of $\int_0^1 x\tan^{-1} x^2\,dx$. **5**

10. Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers. **4**

11. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$. **5**

12. The first two terms of a geometric sequence are $a_1 = p$ and $a_2 = p^2$. Obtain expressions for S_n and S_{2n} in terms of p, where $S_k = \sum_{j=1}^{k} a_j$. **1,1**

Given that $S_{2n} = 65S_n$ show that $p^n = 64$. **2**

Given also that $a_3 = 2p$ and that $p > 0$, obtain the exact value of p and hence the value of n. **1,1**

13. The function $f(x)$ is defined by

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} \qquad (x \neq \pm 1).$$

Obtain equations for the asymptotes of the graph of $f(x)$. **3**

Show that $f(x)$ is a strictly decreasing function. **3**

Find the coordinates of the points where the graph of $f(x)$ crosses

(i) the x-axis and

(ii) the horizontal asymptote. **2**

Sketch the graph of $f(x)$, showing clearly all relevant features. **2**

[Turn over for Questions 14 to 16 on *Page four*

Marks

14. Express $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions. **4**

Hence, or otherwise, obtain the first three non-zero terms in the Maclaurin expansion of $\dfrac{x^2+6x-4}{(x+2)^2(x-4)}$. **5**

15. (*a*) Solve the differential equation

$$(x+1)\frac{dy}{dx} - 3y = (x+1)^4$$

given that $y = 16$ when $x = 1$, expressing the answer in the form $y = f(x)$. **6**

(*b*) Hence find the area enclosed by the graphs of $y = f(x)$, $y = (1-x)^4$ and the x-axis. **4**

16. (*a*) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= 2 \\ -5x + 2y - 4z &= 1. \end{aligned}$$

5

(*b*) Show that the line of intersection, L, of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations

$$\begin{aligned} x &= \lambda \\ y &= 4\lambda - 14 \\ z &= 5\lambda - 20. \end{aligned}$$

2

(*c*) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$. **4**

[*END OF QUESTION PAPER*]